Taxi Routing Optimization

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Mathematical Model of Taxi Routing

Goal of taxi routing:

- Determine the minimum number of taxis needed to serve all the trips without causing delays (with 20/20 hindsight)

Input to the model:

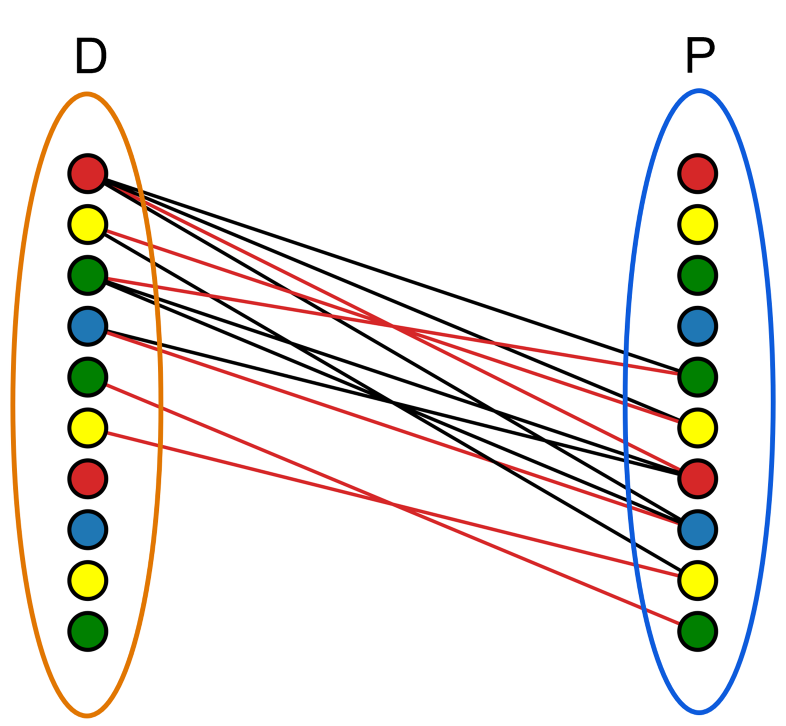
- A collection of n taxi trips $N = \{(p_1, T_1^p, d_1, T_1^d), \cdots, (p_n, T_n^p, d_n, T_n^d)\}$, where the i^{th} trip picks up a passenger at time T_i^p at point p_i and drops off the fare at time T_i^d at point d_i, $i \in \{1, \cdots, n\}$
- Maximum allowable taxi waiting time δ (in minutes)

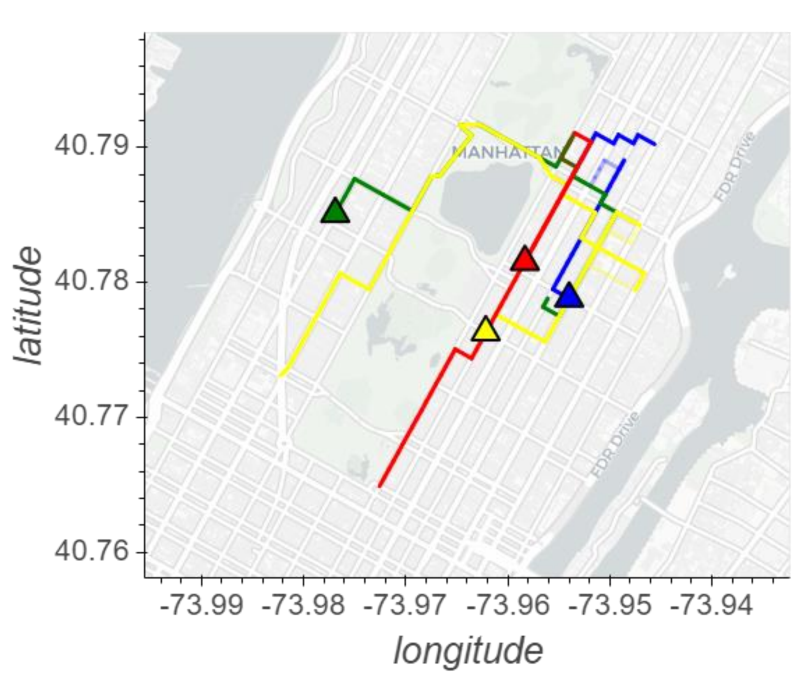
Construct a bipartite graph $G = (D, P, E)$:

- $D = \{(d_1, T_1^d), \cdots, (d_n, T_n^d)\}$ denotes the set of drop-off nodes
- $P = \{(p_1, T_1^p), \cdots, (p_n, T_n^p)\}$ denotes the set of pickup nodes
- $E = \left\{\left\{(d_i, T_i^d), (p_j, T_j^p)\right\} : T_j^p - T_i^d - \delta \leq time(d_i, p_j) \leq T_j^p - T_i^d\right\}$
- Elapsed time between the drop-off time for d_i and the pickup time for p_j, that is, $T_j^p - T_i^d$, is at least the time needed to travel between these two points $time(d_i, p_j)$ and at most $time(d_i, p_j) + \delta$; hence, the taxi can reach the new pickup in time, and does not need to wait more than δ minutes for the pickup to be ready

Lemma Given any matching of size m, there exists a covering of all n trips with $n - m$ taxis

- Application: *Maximum cardinality matching gives rise to the minimum number of taxis that cover all the trips [1][2]*



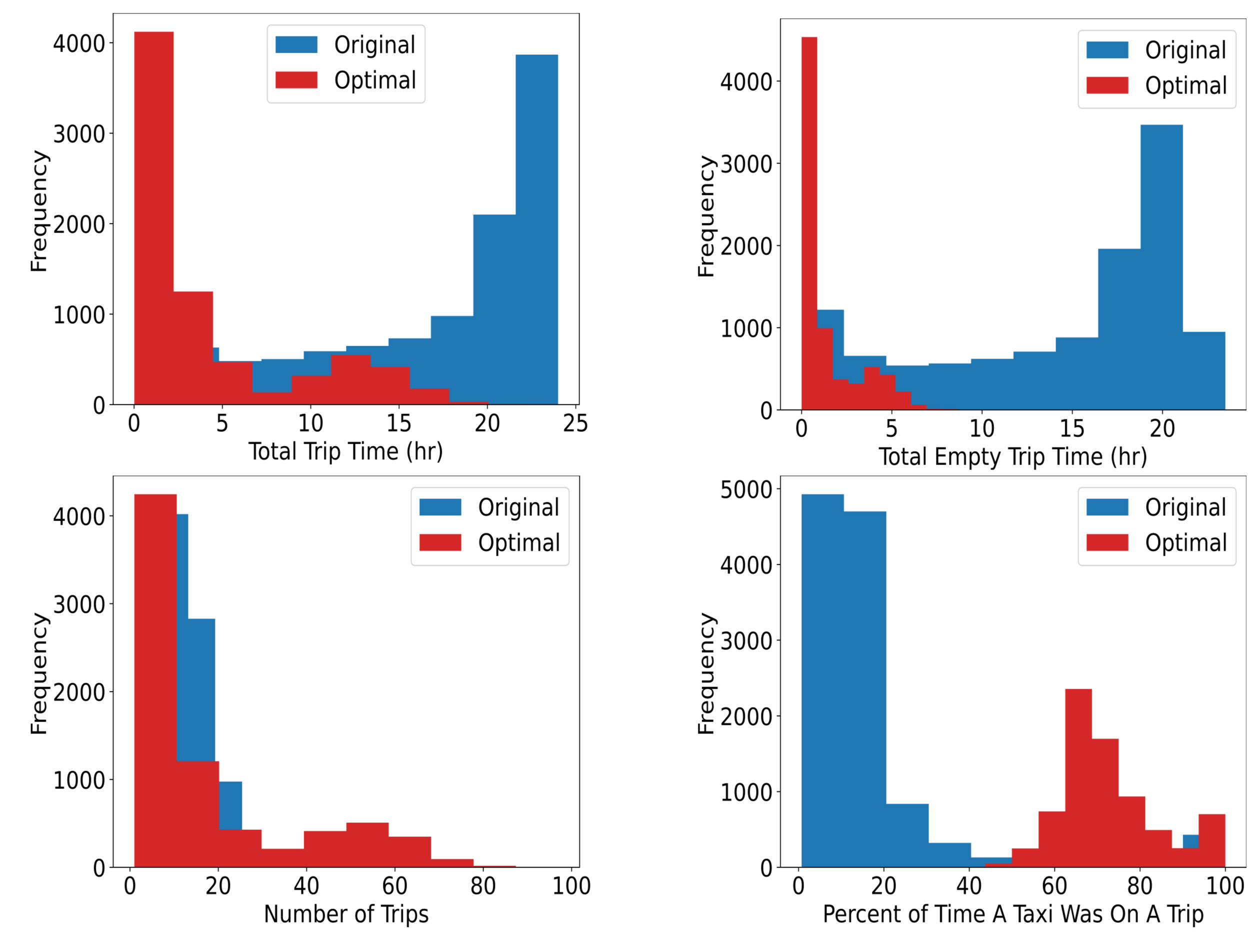


- ❑ An example of 10 trips that can be covered by 4 taxis
- ❑ Maximum cardinality matching ($m = 6$) is colored in red
- ❑ Nodes are colored with respect to their corresponding taxi routes

Computational Results

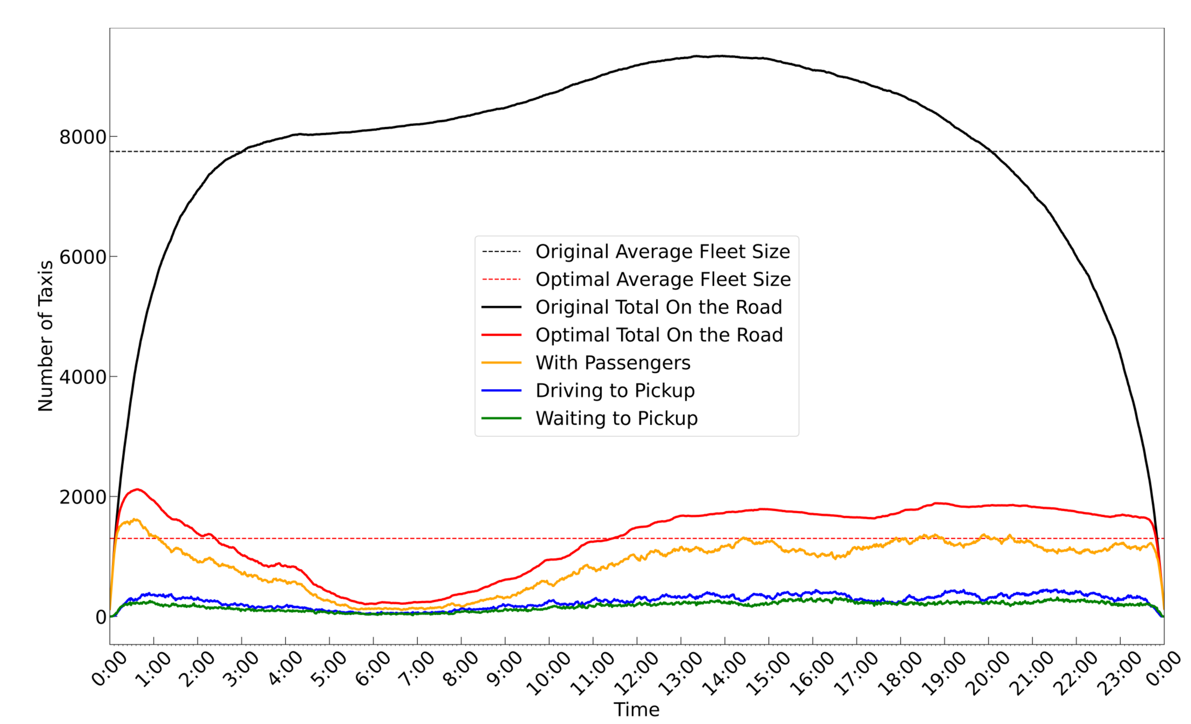
The optimal taxi routing (with δ = 10) is compared with the actual one

- Total number of taxis that covers all trips reduces from 11568 to 7475, a *35%* reduction
- Total trip time and empty trip time are greatly reduced, from 16.1 to 4.2 hours and from 14.3 hours to 1.4 hours, respectively, on average
- To offset the reductions in trip time, the number of trips and on-trip percentage are greatly increased, from 11.3 to 17.5 and from 17.1% to 72.6% respectively, on average



A comparison of the time distribution of the optimal taxi fleets (with breakdown) and the actual taxi fleets

- On average, number of circulating taxis reduces from 7748 to 1300, an *83%* reduction



References

[1] M. M. Vazifeh, P. Santi, G. Resta, S. H. Strogatz & C. Ratti, "Addressing the minimum fleet problem in on-demand urban mobility." *Nature* 557.7706 (2018): 534-538.

[2] X. Zhan, X. Qian & S. Ukkusuri, "A graph-based approach to measuring the efficiency of an urban taxi service system." *IEEE Transactions on Intelligent Transportation Systems* 17.9 (2016): 2479-2489.

Theoretical Results

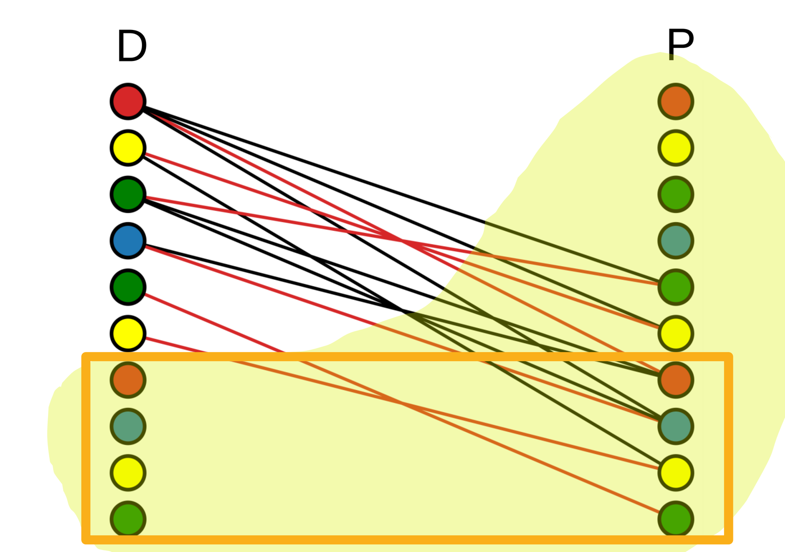
Min-max theorem is an important concept in solvable discrete optimization problems; we prove a min-max theorem for the taxi routing problem

- Consider a simplified setup: $E = \{\{(d_i, T_i^d), (p_j, T_j^p)\} : time(d_i, p_j) \leq T_j^p - T_i^d\}$; i.e., no upper bound on taxi waiting time ($\delta = \infty$)

Definition Any two trips N_i and N_j are considered *compatible* with each other if a taxi can reach from d_i to p_j or from d_j to p_i in time

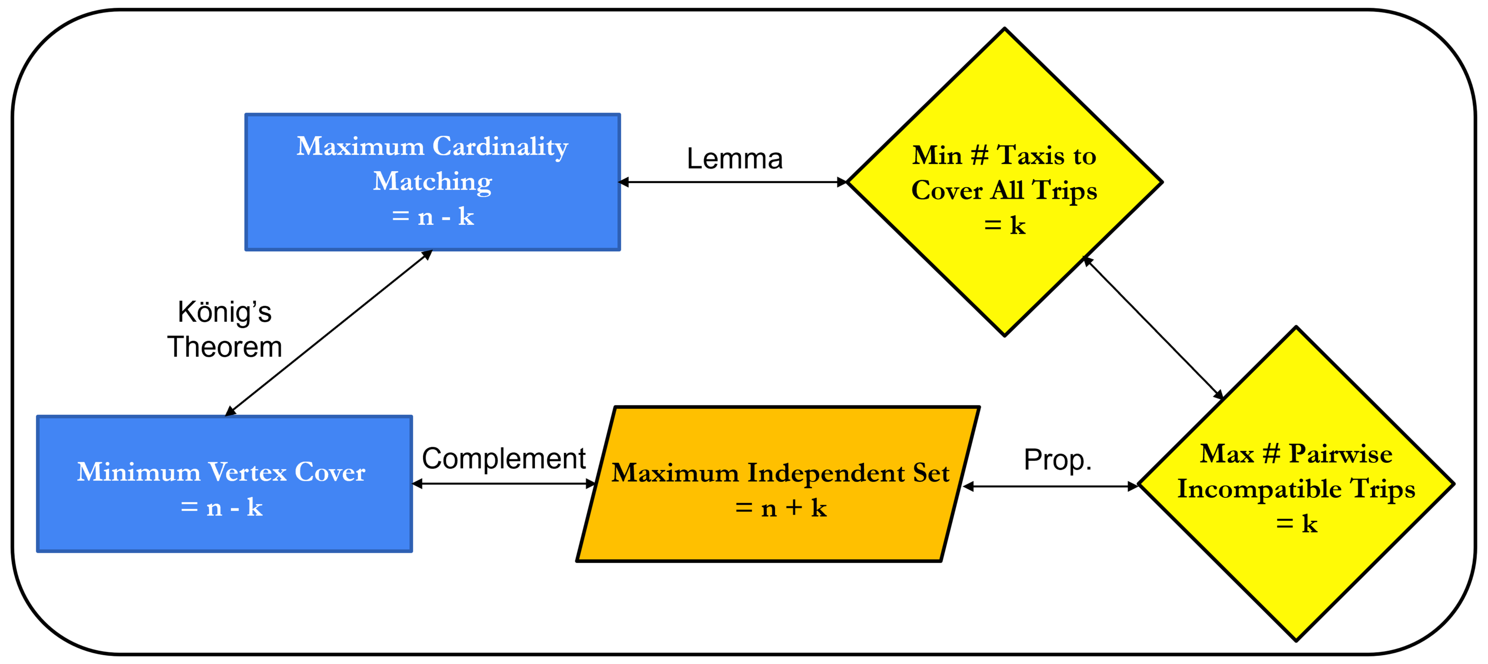
Proposition Consider two disjoint sets $P = \{p_1, \cdots, p_n\}$ and $D = \{d_1, \cdots, d_n\}$ and a subset $I \subseteq P \cup D$, where $|I| = n + k$, for some $k \geq 0$; then there exists a subset $K \subseteq \{1, \cdots, n\}$ where $|K| = k$ and $S = \cup_{i \in K}\{p_i, d_i\} \subseteq I$

- Lower bound certificate: An independent set of size $n + k$ (nodes in shaded region) leads to k trips that are *pairwise incompatible* (orange box)



Theorem
The *maximum size of a set of trips that are pairwise incompatible* is equal to the minimum number of taxis needed to cover all trips

Proof.



Future Work

- The dual object when there's an upper bound (δ) to the waiting time for the new pickup
- An optimal taxi routing that accounts for secondary objectives of waiting time (e.g., minimize maximum waiting time)